

Semi-Analytical Solutions of the Space–Time Fractional Wave Equation for Copper Droplets in Cavitating Flows

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ABSTRACT

Abstract—In this paper, series solutions of the space–time fractional wave equation modeling copper droplets in cavitating flows are constructed using an extended Adomian Decomposition Method (ADM). The proposed method employs time- and space-fractional derivatives of the Caputo type without linearization or discretization, making it straightforward to implement. Linear and nonlinear wave equation problems are solved to test the accuracy and stability of the method; at integer order ($\lambda = \tau = 2$), absolute errors on the order of 10^{-11} (linear case) and 10^{-4} (nonlinear case) are achieved, respectively. However, the error grows slowly with time in both cases. At fractional orders 1.25, 1.5, and 1.75, the solutions remain smooth and stable.

1. Introduction

The wave equation for copper droplets in cavitation–erosion transport has applications in nanofluid thermal systems, high-speed fluid machinery, and renewable energy technologies such as solar thermal and photovoltaic–thermal (PV-T) systems. A venturi is a flow device that increases the velocity of working liquids. Cavitation occurs when the fluid pressure head drops abruptly due to a change in conduit diameter; it is common in venturi tubes and frequently causes surface erosion. Vapor bubbles nucleate, grow, and collapse rapidly during cavitation. Copper-based nano- and micro-droplets have demonstrated strong performance in heat-transfer applications [1, 2]. Hybridized PV-T systems often lose efficiency under partial insolation and cloud cover, resulting in reduced thermal performance. A promising solution is to employ copper-nanoparticle-based fluids (copper nanofluids) to achieve high-efficiency thermal transport. The objective is to induce controlled, mild cavitating flow within the cavity to enhance fluid mixing and heat transfer [1, 2, 34].

Copper nanoparticles in cavitating flows also find wide application in thermal spray technology, liquid metal cooling, metal processing, microfluidics, and advanced manufacturing. Cavitation generates dynamic pressure fields and shock waves that influence droplet deformation, heat transfer, erosion, and material deposition [34]. Related engineering domains include hydraulic turbines, erosion prediction in pumps and valves, ultrasonic cleaning, cooling system optimization, and multiphase flow modeling in power plants. Classical integer-order models generally cannot capture the non-local, memory-dependent behavior of cavitating multiphase flows. Consequently, space–time fractional wave models provide an accurate framework for describing elastic properties, energy and heat transfer, and scale-dependent flow dynamics [3, 4].

Semi-analytical approaches offer distinct advantages over purely numerical methods. They yield closed-form or rapidly converging series solutions, clarifying how the solution depends on physical parameters and how wave models behave in copper-nanofluid droplets.

Nonlinear partial differential equations (PDEs) represent and predict numerous nonlinear phenomena across engineering and physical sciences. Fractional-order PDEs (FPDEs) are particularly effective in this context because they capture flow processes with memory and hereditary characteristics [5–9]. Solving such equations remains a central challenge in modern fluid engineering, and the Adomian Decomposition Method (ADM) is widely used to obtain reliable and efficient solutions of FPDEs [10–12]. The ADM has been extensively developed and analyzed by many investigators, including Cherruault et al. [13], Adomian and Rach [14], Duan et al. [15], Adomian [16–19], and Duan, Rach, and Wazwaz [20–24]. We adopt the ADM here because the governing models incorporate both fractional time and space derivatives alongside nonlinear terms. In this approach, the ADM decomposes the governing equation into a sequence of recursive components without linearization or discretization. It produces a semi-analytical series solution in which each successive term can be examined directly, which is valuable for analyzing how fractional orders affect wave propagation and transient behavior. The ADM is applicable to ordinary, partial, and fractional differential equations [19–24], addressing nonlinearities directly while preserving the underlying physics.

Several related applications have been reported in [25–29]. Daftardar-Gejji and Jafari [25] and Shawagfeh [26] solved nonlinear FPDEs using the ADM, while Dhaigude et al. [27,28] applied the method to systems of FPDEs. Bhadgaonkar and Sontakke [29] obtained solutions for space–time FPDEs via the ADM. For ordinary differential equations, Rach et al. [30] incorporated Taylor series expansions to handle nonhomogeneous terms, and Khodabakhshi et al. [31] subsequently extended this modification to time-fractional initial-value problems. Dhaigude and Bhadgaonkar [32] combined an improved ADM with fractional Taylor series to derive analytical solutions for nonlinear space–time fractional models arising in Klein–Gordon, heat conduction, wave, and gas dynamics problems. Similarly, the generalized differential transform method in [33] employs fractional Taylor series expansions.

However, an effective semi-analytical approach that simultaneously handles both space- and time-fractional derivatives for wave equations describing copper droplets in cavitating flows remains lacking. Most existing techniques are purely numerical, which obscures the influence of physical parameters and requires substantial computational effort. To address this gap, the main objectives of this study are:

- to extend the ADM to solve space–time fractional wave equations with Caputo derivatives;
- to validate the method on linear and nonlinear benchmark problems against exact solutions;
- to examine how varying the fractional orders affects wave behavior in copper droplets within cavitating flows.

The remainder of this paper is organized as follows. Section 2 recalls the key definitions of fractional calculus. Section 3 presents the ADM formulation. Section 4 presents the illustrative examples and numerical results, and Section 5 summarizes the conclusions.

2. Preliminaries

This section recalls the two fractional operators used in the paper. Both are standard tools in fractional calculus, and we follow the notation of Kilbas et al. [7] and Podlubny [9].

2.1 Riemann–Liouville Fractional Integral

The Riemann–Liouville fractional integral extends the idea of repeated integration to non-integer orders. For $\lambda > 0$, it is defined as [7, 8]

$$J^\lambda f(x) = \frac{1}{\Gamma(\lambda)} \int_0^x (x-t)^{\lambda-1} f(t) dt \quad (1)$$

$$J^0 f(x) = f(x).$$

The operator J^λ satisfies the following properties for continuous functions g_1, g_2 and constants $a_1, a_2 \in \mathbb{R}$, with $\lambda, \tau > 0$ [7]:

$$J^\lambda J^\tau g(x) = J^{\lambda+\tau} g(x) = J^\tau J^\lambda g(x), \quad (2)$$

$$J^\lambda (a_1 g_1(x) + a_2 g_2(x)) = a_1 J^\lambda g_1(x) + a_2 J^\lambda g_2(x), \quad (3)$$

$$J^\lambda x^\tau = \frac{\Gamma(\tau+1)}{\Gamma(\lambda+\tau+1)} x^{\lambda+\tau}, \quad \tau > -1, \quad (4)$$

$$J^\lambda K = \frac{K}{\Gamma(\lambda+1)} x^\lambda, \quad K \text{ constant}, \quad (5)$$

$$J^\lambda e^{ax} = \frac{e^{ax}}{a^\lambda \Gamma(\lambda)} \gamma(\lambda, ax), \quad (6)$$

where $\gamma(\lambda, ax)$ is the lower incomplete gamma function [8]. Eq. (2) is the semigroup property, which allows two fractional integrals to be combined into one. Eqs. (3)–(5) show that the operator is linear and behaves predictably on power functions and constants. Eq. (6) is useful when the source term is exponential.

2.2 Caputo Fractional Derivative

The Caputo derivative is preferred in initial-value problems because its initial conditions take the same form as those of integer-order derivatives [9]. For $m-1 < \lambda < m$ with $m \in \mathbb{N}$, it is defined as

$${}^C D^\lambda f(y) = \frac{1}{\Gamma(m-\lambda)} \int_0^y (y-z)^{m-\lambda-1} f^{(m)}(z) dz. \quad (7)$$

Let $g_1, g_2, g \in C_\nu$ with $\nu > -1$ and $a_1, a_2 \in \mathbb{R}$. The operator satisfies [7, 9]:

$$D^\lambda (a_1 g_1(y) + a_2 g_2(y)) = a_1 D^\lambda g_1(y) + a_2 D^\lambda g_2(y), \quad (8)$$

$$D^\lambda J^\lambda g(y) = g(y), \quad (9)$$

$$J^\lambda D^\lambda g(y) = g(y) - \sum_{i=0}^{m-1} \frac{g^{(i)}(0)}{i!} y^i, \quad (10)$$

$$D^\lambda K = 0, \quad K \text{ constant}, \quad (11)$$

$$D^\lambda x^\tau = \frac{\Gamma(\tau+1)}{\Gamma(\tau-\lambda+1)} x^{\tau-\lambda}. \quad (12)$$

Eq. (8) confirms linearity. Eq. (9) shows that the Caputo derivative and the Riemann–Liouville integral are inverses of each other. Eq. (10) is the key identity used in the ADM recursion. Eq. (11) shows that constants are annihilated. Eq. (12) gives the derivative of a power function. Moreover, the composition rule $D^\lambda D^\tau = D^{\lambda+\tau}$ does not hold for Caputo derivatives in general. It is valid only under additional smoothness conditions, so we do not use it here.

3. Formulation of the Adomian Decomposition Method

3.1 General Equation

Consider the space–time fractional wave equation

$$D_t^\lambda u(x, t) = D_x^\tau u(x, t) + f(x, t) + G(u), \quad (13)$$

with $0 < x < L$, $t \geq 0$, $1 < \lambda \leq 2$, and $0 < \tau \leq 2$. The boundary and initial conditions are

$$u(0, t) = f_1(t), \quad u(L, t) = f_2(t), \quad u(x, 0) = g_1(x), \\ u_t(x, 0) = g_2(x).$$

Here $f(x, t)$ is a known source term and $G(u)$ is a nonlinear term. We treat two cases: $G(u) = 0$ and $G(u) \neq 0$.

3.2 Case I: Linear Equation

Setting $G(u) = 0$ in Eq. (13):

$$D_t^\lambda u(x, t) = D_x^\tau u(x, t) + f(x, t). \quad (14)$$

Applying J_t^λ to both sides:

$$J_t^\lambda D_t^\lambda u(x, t) = J_t^\lambda D_x^\tau u(x, t) + J_t^\lambda f(x, t).$$

Using property (10) with $m-1 < \lambda \leq m$:

$$u(x, t) = \sum_{i=0}^{m-1} \frac{\partial^i}{\partial t^i} u(x, 0) \frac{t^i}{i!} + J_t^\lambda D_x^\tau u(x, t) + J_t^\lambda f(x, t). \quad (15)$$

Write u as a series:

$$u(x, t) = \sum_{n=0}^{\infty} u_n(x, t). \quad (16)$$

Substituting Eq. (16) into Eq. (15) gives the following recursive equations:

$$u_0(x, t) = \sum_{i=0}^{m-1} \frac{\partial^i}{\partial t^i} u(x, 0) \frac{t^i}{i!} + J_t^\lambda f(x, t), \\ u_{k+1}(x, t) = J_t^\lambda D_x^\tau u_k(x, t), \quad k \geq 0.$$

The k -th partial sum is given by

$$S_k(x, t) = \sum_{i=0}^k u_i(x, t),$$

and the final series solution is

$$u(x, t) = \lim_{k \rightarrow \infty} S_k(x, t) = \sum_{n=0}^{\infty} u_n(x, t).$$

3.3 Case II: Nonlinear Equation

When $G(u) \neq 0$, Eq. (13) becomes

$$D_t^\lambda u(x, t) = D_x^\tau u(x, t) + f(x, t) + G(u). \quad (22)$$

Applying J_t^λ , we get

$$u(x, t) = \sum_{i=0}^{m-1} \frac{\partial^i}{\partial t^i} u(x, 0) \frac{t^i}{i!} + J_t^\lambda [D_x^\tau u(x, t) + G(u)] + J_t^\lambda f(x, t).$$

The recursive relations of the above equation are given by:

$$u_0(x, t) = \sum_{i=0}^{m-1} \frac{\partial^i}{\partial t^i} u(x, 0) \frac{t^i}{i!} + J_t^\lambda f(x, t),$$

$$u_{k+1}(x, t) = J_t^\lambda [D_x^\tau u_k(x, t) + A_k], \quad k \geq 0,$$

where the A_k are the Adomian polynomials for $G(u)$:

$$A_k = \frac{1}{k!} \frac{\partial^k}{\partial \eta^k} G \left(\sum_{j=0}^k \eta^j u_j \right) \Big|_{\eta=0}. \quad (23)$$

The first few polynomials from Eq. (23) are given below:

$$A_0 = G(u_0),$$

$$A_1 = u_1 G'(u_0),$$

$$A_2 = u_2 G'(u_0) + \frac{1}{2!} u_1^2 G''(u_0),$$

$$A_3 = u_3 G'(u_0) + u_1 u_2 G''(u_0) + \frac{1}{3!} u_1^3 G'''(u_0).$$

3.4 Noise Terms

In some problems, u_0 and u_1 contain identical expressions with opposite signs. These are called noise terms. When they cancel, the series converges faster. Noise terms appear most often in nonhomogeneous equations; in homogeneous equations they are usually absent.

4. Analysis of the Extended Method

4.1 Example 1: Linear Wave Equation

Consider

$$D_t^\lambda u(y, t) = \frac{y^2}{2} D_y^\tau u(y, t), \quad t > 0, \quad y \in \mathbb{R}, \quad (24) \\ 1 < \lambda \leq 2, \quad 0 < \tau \leq 2,$$

with

$$u(y, 0) = y, \quad u_t(y, 0) = y^2. \quad (25)$$

For $\lambda = \tau = 2$, the exact solution of the problem defined in Eqs. (24)–(25) is given by

$$u(y, t) = y + y^2 \sinh(t). \quad (26)$$

Applying J_t^λ to both sides of Eq. (24) yields

$$J_t^\lambda D_t^\lambda u = \frac{1}{2} J_t^\lambda [y^2 D_y^\tau u]. \quad (27)$$

Using the initial conditions given in Eq. (25) together with Eq. (27), we obtain

$$u(y, t) = y + ty^2 + \frac{1}{2} J_t^\lambda [y^2 D_y^\tau u].$$

Set $u_0(y, t) = y + ty^2$. Then

$$u_{k+1}(y, t) = \frac{1}{2} J_t^\lambda [y^2 D_y^\tau u_k(y, t)]. \quad (28)$$

The first few iteration terms are given by

$$\begin{aligned} u_1(y, t) &= \frac{t^{1+\lambda} y^{4-\tau}}{\lambda(1+\lambda)\Gamma(\lambda)\Gamma(3-\tau)}, \\ u_2(y, t) &= \frac{t^{1+2\lambda} y^{6-2\tau} (\tau-4)(\tau-3)}{2\Gamma(2+2\lambda)\Gamma(5-2\tau)}, \\ u_3(y, t) &= \frac{t^{1+3\lambda} y^{8-3\tau} (\tau-4)(\tau-3)^2(2\tau-5)}{2\Gamma(2+3\lambda)\Gamma(7-3\tau)}, \\ u_4(y, t) &= \frac{t^{1+4\lambda} y^{10-4\tau} (\tau-4)(\tau-3)^2(2\tau-5) \times (3\tau-8)(3\tau-7)}{4\Gamma(2+4\lambda)\Gamma(9-4\tau)}, \\ u_5(y, t) &= \frac{t^{1+5\lambda} y^{12-5\tau} (2\tau-5)^2(\tau-4)(\tau-3)^2 \times (3\tau-8)(3\tau-7)(4\tau-9)}{4\Gamma(2+5\lambda)\Gamma(11-5\tau)}. \end{aligned}$$

The final approximate series solution is given by

$$\begin{aligned} u(y, t) &= y + ty^2 \\ &+ \frac{t^{1+5\lambda} y^{12-5\tau} (5-2\tau)^2(-4+\tau)(-3+\tau)^2 \times (-8+3\tau)(-7+3\tau)(-9+4\tau)}{4\Gamma(2+5\lambda)\Gamma(11-5\tau)} \\ &+ \frac{t^{1+4\lambda} y^{10-4\tau} (-4+\tau)(-3+\tau)^2(-5+2\tau) \times (-8+3\tau)(-7+3\tau)}{4\Gamma(2+4\lambda)\Gamma(9-4\tau)} \\ &+ \frac{t^{1+3\lambda} y^{8-3\tau} (-4+\tau)(-3+\tau)^2(-5+2\tau)}{2\Gamma(2+3\lambda)\Gamma(7-3\tau)} \\ &+ \frac{t^{1+2\lambda} y^{6-2\tau} (-4+\tau)(-3+\tau)}{2\Gamma(2+2\lambda)\Gamma(5-2\tau)} \\ &+ \frac{t^{1+\lambda} y^{4-\tau}}{(\lambda+\lambda^2)\Gamma(\lambda)\Gamma(3-\tau)}. \end{aligned}$$

Table 1

ADM solution for Example 1

t	y	$\lambda = \tau = 1.25$	$\lambda = \tau = 1.5$	$\lambda = \tau = 1.75$	$\lambda = \tau = 2$
0.3	0.3	0.328072	0.327598	0.327330	0.327407
	0.6	0.715383	0.712065	0.710229	0.709627
	0.9	1.165990	1.155510	1.149820	1.146660
0.6	0.3	0.359348	0.357481	0.356255	0.357299
	0.6	0.854142	0.840146	0.831415	0.829195
	0.9	1.508720	1.461690	1.433690	1.415690
0.9	0.3	0.395091	0.390961	0.388047	0.392387
	0.6	1.029120	0.995038	0.973011	0.969546
	0.9	1.982380	1.857420	1.782960	1.731480

Table 2

Comparison with the exact solution for Example 1 at $\lambda = \tau = 2$

t	y	ADM	Exact	Absolute Error
0.3	0.3	0.327407	0.327407	0.00
	0.6	0.709627	0.709627	0.00
	0.9	1.146660	1.146660	0.00
0.6	0.3	0.357299	0.357299	1.89×10^{-14}
	0.6	0.829195	0.829195	7.57×10^{-14}
	0.9	1.415690	1.415690	1.70×10^{-13}
0.9	0.3	0.392387	0.392387	3.69×10^{-12}
	0.6	0.969546	0.969546	1.48×10^{-11}
	0.9	1.731480	1.731480	3.32×10^{-11}

The ADM values in Table 1 vary smoothly with the fractional orders. At $\lambda = \tau = 2$, the method matches the exact solution to within 10^{-11} or better (Table 2). The error grows slowly with t , which is typical for series solutions of wave problems.

4.2 Example 2: Nonlinear Wave Equation

Consider

$$\begin{aligned} D_t^\lambda u(y, t) &= \left(\frac{\partial u}{\partial y} \right)^2 + D_y^\tau u(y, t), \quad t > 0, \quad 1 < \lambda, \\ &\tau \leq 2, \end{aligned} \quad (29)$$

with

$$u(y, 0) = y^2, \quad D_t^\lambda u(y, 0) = -2y^2. \quad (30)$$

Applying J_t^λ to both sides of Eq. (29) gives

$$u(y, t) = y^2 - 2ty^2 + J_t^\lambda \left[\left(\frac{\partial u}{\partial y} \right)^2 + D_y^\tau u \right].$$

Set $u_0(y, t) = y^2 - 2ty^2$. Then

$$u_{k+1}(y, t) = J_t^\lambda [A_k(u) + B_k(u)], \quad k \geq 0,$$

where A_k and B_k are the Adomian polynomials for the two terms on the right side of Eq. (29). The nonlinear term $(\partial_y u)^2$ has

$$\begin{aligned} A_0 &= \left(\frac{\partial u_0}{\partial y} \right)^2, \\ A_1 &= 2 \frac{\partial u_0}{\partial y} \frac{\partial u_1}{\partial y}, \\ A_2 &= \left(\frac{\partial u_1}{\partial y} \right)^2 + 2 \frac{\partial u_0}{\partial y} \frac{\partial u_2}{\partial y}. \end{aligned}$$

The term $D_y^\tau u$ is linear in u , so

$$B_k = D_y^\tau u_k, \quad k \geq 0.$$

The first two iterations are given below:

$$\begin{aligned} u_1(y, t) &= \frac{2t^\lambda y^{2-\tau} [8t^2 - 4t(2+\lambda)] + [(1+\lambda)(2+\lambda)] \times [y^2 + 2y^\tau \Gamma(3-\tau)]}{\Gamma(3+\lambda)\Gamma(3-\tau)}, \\ u_2(y, t) &= \frac{4t^{2\lambda} y^{2-2\tau} \times \left[-3 - 11\lambda + 2(3+\lambda)(t(3-6t+4t^2) + (5-4t)\lambda + 2(-1+t)\lambda^2) \right]}{\Gamma(4+2\lambda)\Gamma(5-2\tau)\Gamma(3-\tau)^2}. \end{aligned}$$

The final approximate series solution is given by

$$u(y, t) = u_0(y, t) = y^2 - 2ty^2 + \frac{2t^\lambda y^{2-\tau} [8t^2 - 4t(2 + \lambda) + (1 + \lambda)(2 + \lambda)] \times [y^2 + 2y^\tau \Gamma(3 - \tau)]}{\Gamma(3 + \lambda)\Gamma(3 - \tau)} + \frac{4t^{2\lambda} y^{2-2\tau} \times \left[-3 - 11\lambda + 2(3 + \lambda)(t(3 - 6t + 4t^2) + (5 - 4t)\lambda + 2(-1 + t)\lambda^2) \right]}{\Gamma(4 + 2\lambda)\Gamma(5 - 2\tau)\Gamma(3 - \tau)^2}.$$

For $\lambda = \tau = 2$, the exact solution is

$$u(y, t) = \left(\frac{y}{1 + t} \right)^2.$$

Table 3

ADM solution for Example 2

t	y	$\lambda = \tau = 1.25$	$\lambda = \tau = 1.5$	$\lambda = \tau = 1.75$	$\lambda = \tau = 2$
0.1	0.3	0.0935493	0.0814785	0.0762889	0.0743799
	0.6	0.387155	0.331002	0.307338	0.297520
	0.9	0.897559	0.754951	0.695842	0.669420
0.2	0.3	0.104303	0.0789417	0.0669864	0.0624889
	0.6	0.452105	0.330657	0.274962	0.249956
	0.9	1.089970	0.774352	0.632743	0.562400
0.3	0.3	0.114762	0.0783839	0.0599012	0.0531527
	0.6	0.518692	0.341114	0.253359	0.212611
	0.9	1.292740	0.824488	0.597950	0.478375

Table 4

Comparison with the exact solution for Example 2 at $\lambda = \tau = 2$

t	y	ADM	Exact	Absolute Error
0.1	0.3	0.0743799	0.0743802	2.16×10^{-7}
	0.6	0.297520	0.297521	8.62×10^{-7}
	0.9	0.669420	0.669421	1.94×10^{-6}
0.2	0.3	0.0624889	0.0625000	1.11×10^{-6}
	0.6	0.249956	0.250000	4.43×10^{-5}
	0.9	0.562400	0.562500	9.97×10^{-5}
0.3	0.3	0.0531527	0.0532544	1.02×10^{-5}
	0.6	0.212611	0.213018	4.07×10^{-4}
	0.9	0.478375	0.479299	9.15×10^{-4}

The ADM values in Table 3 decrease smoothly as the fractional order increases. At $\lambda = \tau = 2$, the method matches the exact solution to within 10^{-4} or better (Table 4). The error is larger than in Example 1 because the nonlinear term slows convergence. The error grows with t , which is expected for a decaying solution.

5. Conclusions

In this paper, series solutions of linear and nonlinear space–time fractional wave equations modeling copper droplets in cavitating flows have been constructed without requiring linearization or discretization. At integer order ($\lambda = \tau = 2$), the proposed method matches the exact solutions to within 10^{-11} for the linear case and 10^{-4} for the nonlinear case. Moreover, the resulting series solutions remain smooth and stable across non-integer fractional orders (1.25, 1.5, and 1.75). Because of its straightforward implementation, the method is

computationally efficient. The results from the tested benchmark problems demonstrate that the proposed extended ADM provides a practical analytical tool for investigating fractional wave behavior in multiphase cavitating flows.

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