

Development of a Mathematical Equation for Sustainable Design of Roof Profile of High Head Orifice Spillway

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ABSTRACT

Large orifices play a significant role in the sustainability of dams, in the release of water, and in the management of sediments during reservoir operation. Numerous studies have been conducted to analyze discharge and other hydraulic coefficients, but few studies have examined the design of roof profiles for orifices in large hydraulic structures. This experimental study has been used to develop the equation for the roof profile and to investigate the impact on the coefficient of discharge after the fixation of the bottom and roof profiles under varying gate openings and different heads. The minimum difference between the bottom nappe profile of the physical model and the trajectory equation is sensitive to various factors. Moreover, significant deviations have been observed in the upper profiles compared with existing United States Bureau of Reclamation (USBR) equations and those developed by other researchers. These investigations have further revealed that after fixing a solid bottom profile, the discharge coefficient for a sharp-edged orifice. Therefore, the existing equation for the roof profile in the literature has been improved to develop a new equation with greater scope for applicability. It has been observed that using a solid roof profile, designed with an improved equation, significantly increased the coefficient of discharge from 0.66 to 0.952. The proposed equation is valid for heads between 40–80 m and for the tested discrete gate openings (10, 12, 14, and 16 m) under the present experimental conditions.

1. Introduction

An orifice is an important component designed for a particular dam and used to release water from the reservoir in a controlled manner. Large rectangular orifices are often built to release excess water from the reservoir and manage sediment. The release of huge volumes of water from reservoirs with higher velocities makes the design of orifices more complex for passing over the crest of orifice spillways. Therefore, detailed investigations of orifice spillways are required at the feasibility stage of the dam project,

and the use of scaled physical models is to be mandated to carry out hydraulic analyses [1,2,3]. Moreover, such spillways are rather difficult to operate at high discharges due to turbulence, sediment transport [4,5], the formation of natural bed structures [6], and the risk of cavitation [6]. The design parameters of an orifice are affected by a variety of factors, among which the water head is the most critical [7]. Moreover, due to variability in the discharge coefficient (C_d), the use of physical models is crucial for accurately simulating orifice spillway

flows. Furthermore, the dimensions of the orifices vary from case and should be adjusted in light of the dam's operational and structural requirements [9,10,11].

Several studies have been conducted to determine hydraulic coefficients, including the discharge coefficient (Cd), for assessing design and smooth operation in the control and regulation of flow through orifices [12,13]. Essien et al. [14] conducted experimental work on the discharge coefficient (Cd) and developed an empirical correlation between orifice size and Cd. The bottom and roof profiles of orifice spillways maximise discharge and enable controlled operations. The roof profile determines water movement, whereas the bottom profile determines water exit behavior. However, the roof profile design has not yet been standardized [15]. Vatankhah et al. [16] performed 588 laboratory tests to analyze the elliptical side-orifice, using experimental data to calibrate theoretical models under free-outflow conditions. Bhattarai and Sharma [9] highlighted that the size of the orifice affects the flow capacity of spillways significantly in embankment dams. Similarly, Jothiprakash et al. [17] observed that upstream flow conditions and the geometry of the orifice have a significant impact on the coefficient of velocity. Collectively, all these studies highlight the crucial role of orifice geometry, hydraulic coefficients, and profile design in improving the efficiency and performance of orifice spillways.

A detailed literature review of spillways highlights the importance of safely discharging floods and emphasizes the need to study the bottom and roof profiles. This is required in addition to the discharge coefficient for effective design and safe operation of orifice spillways. In comparison, Gadge et al. [18] developed guidelines for orifice design and equations for roof and bottom profiles. However, there is limited literature on the design of these profiles for higher heads and larger orifice openings. To fill this gap, the present study has been planned, conducted, and investigated through the scaled-down physical model to analyze flows under varying gate openings (10 to 16m) and water heads (40 to 80 m). An equation has been proposed through this study as an empirical design guideline within the tested range, rather than for a universal standard. The findings of this study may be helpful to project managers and design engineers in the sustainable design of orifice spillways and in enhancing their understanding of large orifice spillways.

2. Materials and Methods

Therefore, in this study, a physical model at a 1:100 scale was developed to examine the upper and lower profiles of a large-orifice-type spillway.

In dimensional analysis, we are concerned only with the type or nature of physical quantities, such as mass (M), length (L), and time (T), and not with their numerical values.

Before the physical construction of any hydraulic structure, a dimensionless analysis was performed to determine the key hydraulic parameters governing the behavior of the flow profiles. In the present study, which pertains to the roof profile of the orifice spillway, the Buckingham Pi-theorem was applied to conduct dimensional analysis. The Pi-Theorem simplifies the complicated physical problems by classifying physical quantities into dimensionless groups. The methodology was prepared to conduct the experiments, as outlined in the flow chart in Figure 1. Following the dimensional analysis, the model was constructed as shown in Figure 2. The Froude No. plays a key role in dimensional analysis and is simplified when gravitational forces are dominant in fluid flow problems. Moreover, Froude No. ensures dynamic similarity between flows in model and prototype, being dominated by gravity and inertia, whereas other forces, such as viscosity and capillarity, are negligible in such cases. Therefore, Froude similarity is used with the geometric scale, and the selection is made carefully to minimize the impact of dissimilarities in the Reynolds and Weber numbers. In addition, to ensure the reliability of model results, the adopted Reynolds number is kept sufficiently high to maintain fully turbulent flow conditions. As noted in the literature, many researchers suggest that the Reynolds number should exceed 10^5 . In this way, the adequacy of the flow regime is ensured. In the present study, the calculated Reynolds number (approximately 10×10^6) significantly exceeds the commonly recommended minimum criterion ($Re > 10^5$). Thus, the suitability of the flow regime is confirmed. Froude similarity was prioritized in the current investigation, whereas effects of viscosity and surface tension were kept secondary. Additionally, air entrainment and Weber effects are not explicitly modeled.

2.1 Development of physical model

The physical model is built in the hydraulic laboratory of the Centre of Excellence in Water Resources

(CEWRE) at the University of Engineering & Technology, Lahore, as illustrated in Figure 2.

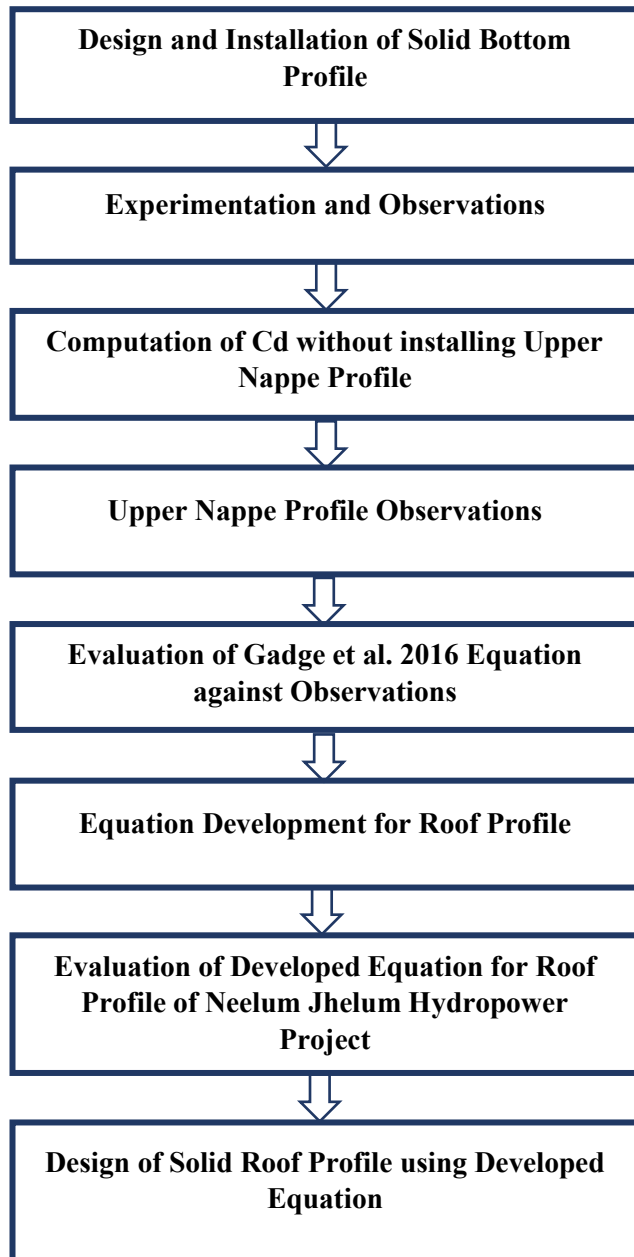


Fig. 1. Methodological Flow Chart



Fig. 2. Constructed scale-down model of orifice spillway in lab environment.

3. Results

3.1 Nappe Profiles of Orifice Flows

To design solid profiles of orifice spillways, observation data of profiles of upper and lower nappe are examined and compared with profiles obtained using empirical equations (2) and (3).

3.2 Bottom Profiles for Sharpe-Edge Large Orifice

Figures 3 and 4 compare the lower profiles from the current scale-down model with profiles generated using the empirical equation for gate-opening ranges of 10-16 m. The experimental observation data from the physical model showed strong agreement with the trajectory profile equation ($X^2 = 4yh_c$). However, minor deviations were observed relative to the empirical results, which can be attributed to uncertainties in qualitative data and to scaling effects inherent in the model. These minor discrepancies highlight the challenges in replicating real-world conditions in a scaled physical model.

$$X^2 = kyh_c \quad (1)$$

The coefficient 'k' in equation (1) plays a crucial role in determining the profile shape, which is more of a parabolic form. Based on previous research reviews, the velocity coefficient for flows through sharp-edged orifices has been found to be near unity, indicating that the flow remains streamlined through the orifice [19], [20]. Consequently, equation (2) is applied to compare the bottom profiles derived from the experimental results in this study. This comparison helps validate the experimental data by analysing the consistency between the observed and theoretical bottom profile shapes, which is expressed as.

$$X^2 = 4yh_c \quad (2)$$

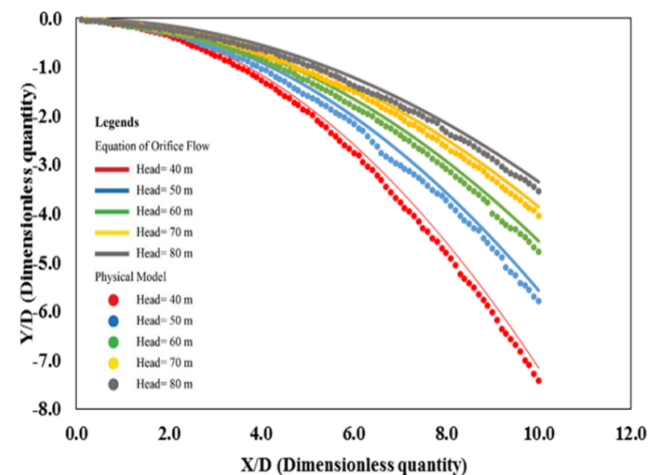


Fig. 1. Comparison of bottom profiles generated through the equation and physical observations against various heads for a gate opening of 10 m

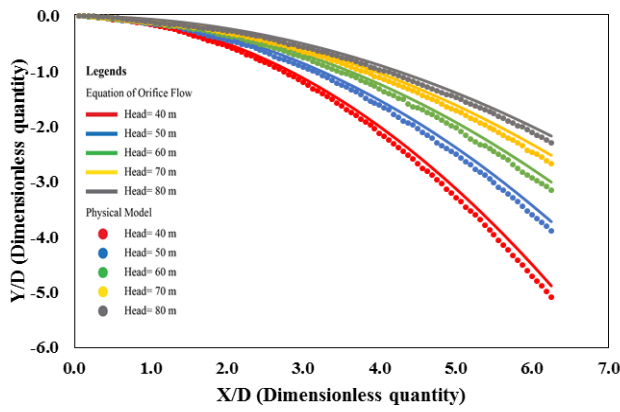


Fig. 2. Comparison of bottom profiles generated through the equation and physical observations against various heads for a gate opening of 16 m

3.3 Roof Profile for Sharpe-Edge Large Orifice

Comparison was made with the physical model observations of roof profiles with the USBR equation [21, 22, 23]. The profiles of upper nappe curves are generated against different water head ranges (40 - 80 m) and various gate operation ranges (10 - 16m). Comparisons of profiles generated using the United States Bureau of Reclamation (USBR) equation (3) revealed that these profiles were elliptical in shape and deviated from the observed profile curves.

$$\frac{x^2}{D^2} + \frac{y^2}{(0.32D)^2} = 1 \quad (3)$$

In the above equation, the coordinates of elliptical profiles are x and y, and the gate opening is denoted by D.

The observed upper nappe profiles were then compared with those generated using Gadge et al.'s (2019) equation (4), accounting for differences between the roof profiles of the current investigation and those of the USBR technique. It was discovered that the development of lower heads was the reason for the unsatisfactory results. As a result, profiles cannot be used directly in this study to ascertain an orifice spillway's roof profile. To develop the roof profile, Equation 4 is modified in the current study.

$$x_1 = a \left(\frac{y_1}{b} \right)^m \quad (4)$$

$$a = A d \left(\frac{h_d}{d} \right)^B \quad (4.1)$$

Where A, B, and m are the coefficients, h_d = design head, $b = 0.1$ to $0.4d$, 'd' can be derived using the expression $D = 1.1$ to $1.4d$, and D = Gate opening.

3.4 Development of Mathematical Equation for Roof Profile Design

Observations made for the roof profile (x-y coordinates) were used to modify Equation 4. Minitab software version 22 was used to perform the stepwise regression analysis to introduce the constant in Equation 4. First, a non-linear regression analysis was performed with a constant m. It was found that the value of R^2 was too low. Therefore, Equation (4) was converted to a linear form using the natural logarithm (ln) to perform the stepwise regression analysis. Overall, the 2,100 combinations of observed values from the physical model of upper nappe profiles were divided into a training dataset (75%) and a testing dataset (25%). Moreover, during the development of the equation, a 90% confidence level was selected, with $\alpha = 0.10$. The coefficient of determination (R^2) for training was 0.816 for the developed Equation (5).

$$x_1 = c_1 a^{c_2} \left(\frac{y_1}{b} \right)^{c_3} \quad (5)$$

$$a = A d \left(\frac{h_d}{d} \right)^B \quad (5.1)$$

Where $c_1 = 29.99408$, $c_2 = 0.8421$, and $c_3 = 1.5766$ are constant.

The developed equation was tested against observed datasets for 12 m and 14 m orifice openings with varying head (40 m to 80 m). The R^2 value was calculated to assess the performance of the developed equation, and it was 0.9227 for a 12 m opening (Figure 5a) and 0.8943 for a 14 m opening (Figure 5b).

The developed equation was tested against the existing roof profile of the Neelum Jehlum Hydropower Project. A comparison was made between the roof profile designed by the Consultants for the Neelum Jehlum Hydropower Project and the roof profile obtained from the developed equation of this model study to validate the general applicability of the equation. Comparative analysis demonstrated that the Consultant-developed roof profile exhibits a flatter, narrower opening than the profile derived from the developed equation. The roof profile from the developed equation shows a steeper, wider opening. The Coefficient of Discharge 'Cd' value of Neelum Jehlum Hydropower Project is 0.90, which could have been improved if this equation were available to the designers at the time of construction of the aforementioned project. The proposed equation is valid for heads between 40–80 m and for the tested discrete gate openings (10, 12, 14, and 16 m) under the present experimental conditions.

4. Discussion

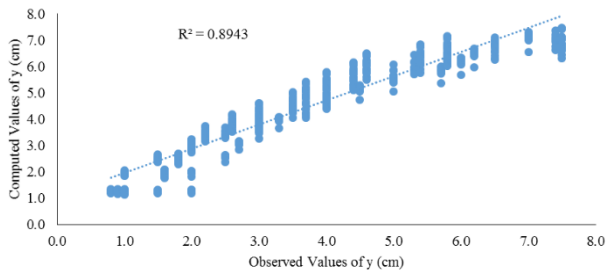
4.1 Impact of Solid Bottom Profile on Cd

The solid bottom profiles play a vital role in enhancing the orifice's discharge coefficient. As shown in Table 1, the discharge coefficient improves significantly with the addition of the solid bottom. The coefficient of discharge ranges from 0.39 to 0.74. This is a higher range than that which was observed without fixing the solid bottom profile.

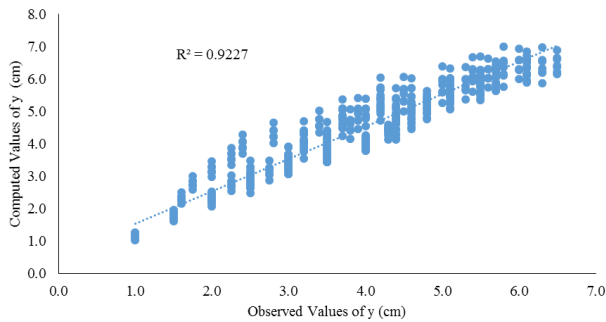
Table 1.

Coefficient of Discharge with a solid bottom

Gate Opening (m)	Coefficient of Discharge					Average Values
	H40 m	H50 m	H60 m	H70 m	H80 m	
10	0.39	0.40	0.43	0.44	0.46	0.42
16	0.63	0.64	0.66	0.69	0.74	0.67



(a)



(b)

Fig. 5. Plot of Observed values vs Computed values to determine different values of R^2 (a) for Gate opening 12 m (b) for Gate opening 14 m

4.2 Effect of a solid roof profile on the "Cd" coefficient of discharge.

The Coefficient of discharge (C_d) for this study was calculated as the observed flow rate divided by the theoretical flow rate from the formula. Initially, the Coefficient of discharge (C_d) was found as 0.66. An improvement in the coefficient of discharge (C_d) to 0.74 was observed after installing a solid bottom profile in the physical model. After installing the solid roof profile developed using Equation 5, the results

showed that the Coefficient of Discharge ' C_d ' of the orifice spillway further improved significantly, as depicted in Figure 6 with an average C_d value of 0.952.

The performance of the solid roof profile obtained through the developed equation was evaluated in terms of the Coefficient of Discharge (C_d), as shown in Figure 6. It is observed that the Coefficient of Discharge (C_d) increases with head while the gate opening remains unchanged, i.e., 16 m. It was also noted that there is a significant increase in the values obtained for the Coefficient of Discharge (C_d) following the installation of a solid roof profile, compared with the Coefficient of Discharge (C_d) values obtained before installation.

The results after installation of the solid roof profile also confirm that the pressures observed along the solid roof profile remain positive, eliminating the risk of cavitation failure. Overall, the designed roof profile, developed using Equation 5, is deemed acceptable based on its pressure and performance, as measured by the Coefficient of Discharge ' C_d '.

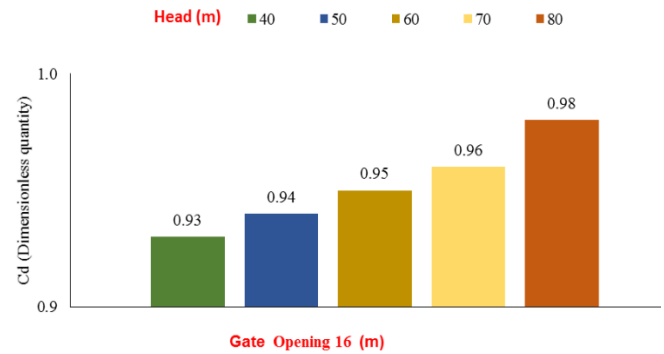


Figure 6. Coefficients of discharge after installation of the solid Roof Profile at various heads with 16 m gate opening.

Comparing the profiles generated using the USBR equation revealed that they were elliptical in shape and deviated from the observed solid-roof profile curves because the USBR equation was developed without accounting for head variation. Similarly, the observed upper nappe profiles differed from those derived from Gadge et al.'s (2019) equation. This was due to the obvious reasons: in the present study, the profiles were drawn for heads ranging from 40 m to 80 m, and the Gadge et al. (2019) equation was developed for heads ranging from 30 to 70 m. Therefore, the variation of profiles observed in both cases under the present scenario was justified, and there could be no comparison between the profiles drawn for two different sets of heads.

5. Conclusion

In the present study, a scaled-down model was constructed in the hydraulic laboratory of CEWRE to

analyze the hydraulic behavior of flow through sharp-edged large orifices. In the present study, under the given operational conditions, the model was tested at 10 m and 16 m gate openings and varying water heads (40 to 80 m). The following conclusions are depicted after conducting the analysis:

1. The solid bottom profile contributed to an improvement in the discharge coefficient, which improved significantly from 0.66 to 0.74.
2. This research has provided design guidelines for the solid roof profile of orifice spillways by investigating profiles of upper nappe under various heads and orifice openings.
3. This study proposed an equation that is valid for heads between 40 – 80 m and for the tested discrete gate openings (10, 12, 14, and 16 m) under the present experimental conditions.
4. The proposed equation is intended as empirical design guidance within the tested range, rather than a universal standard.
5. The solid roof profile installation yielded an average coefficient of discharge of 0.952, and the solid bottom profile coefficient of discharge of 0.74, with no negative pressure along the solid roof profile.
6. After installation of the solid roof profile, qualitative analysis confirmed smooth flow through the orifice without any separation at the designed operating conditions.

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